13 Gravitational fields

13.1 Gravitational field

Candidates should be able to:

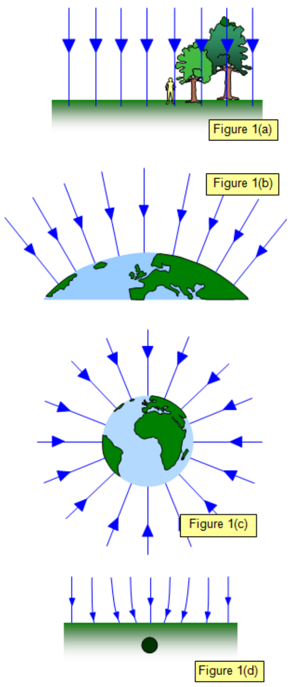
1 understand that a gravitational field is an example of a field of force and define gravitational field as force per unit mass

2 represent a gravitational field by means of field lines

- When **two or more masses** are in proximity with each other there is an **attractive force** between them.
- This force is called **gravity**
- A **gravitational field** is defined as **a region of space where a mass experience a force due to the gravitational attraction of another mass.**
- The SI unit for gravitational field strength is **N kg^{-1}** or **ms^{-2}**
- The gravitational field strength (g) at a point is the force due to gravity or weight (F_g) per unit mass (m) of an object at that point:

$$g = \frac{Fg}{m}$$

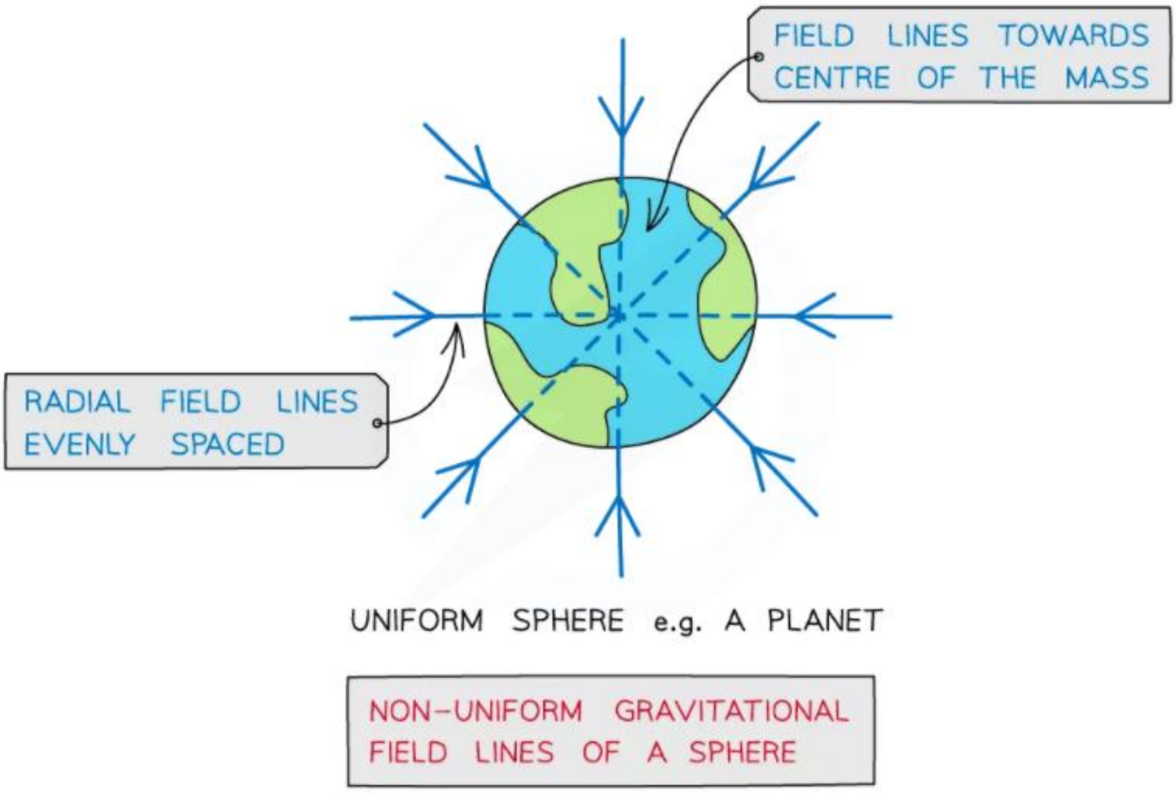
- The larger the planet, the larger the g!
- Gravitational field lines like magnetic and electric field lines gives us an indication to their direction.
- Unlike the other two however, gravitational field lines are always attractive and never repulsive!
- Below are some examples of gravitational field lines



13.2 Gravitational force between point masses

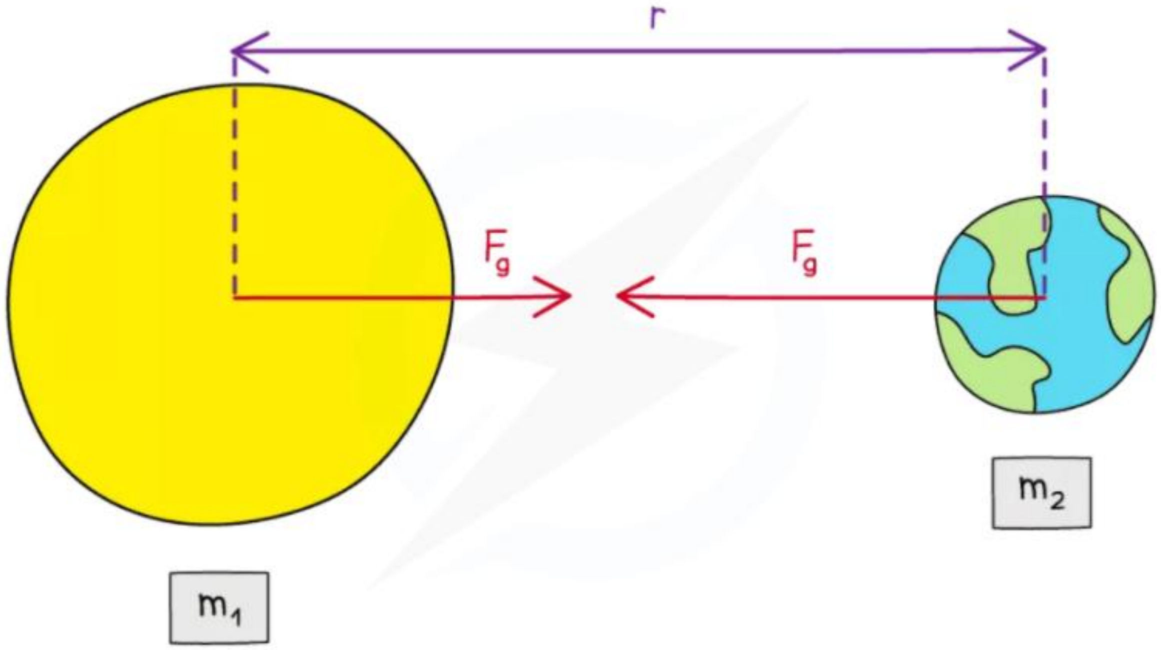
Candidates should be able to:	
1	understand that, for a point outside a uniform sphere, the mass of the sphere may be considered to be a point mass at its centre
2	recall and use Newton's law of gravitation $F = Gm_1m_2/r^2$ for the force between two point masses
3	analyse circular orbits in gravitational fields by relating the gravitational force to the centripetal acceleration it causes
4	understand that a satellite in a geostationary orbit remains at the same point above the Earth's surface, with an orbital period of 24 hours, orbiting from west to east, directly above the Equator

- For a point outside a uniform sphere, the mass of the sphere may be considered to be a point mass at its centre.
- A uniform sphere is one where its mass is **distributed evenly**.
- The gravitational field lines around a uniform sphere are therefore **identical to those around a point mass**
- An object can be regarded as point mass **when a body covers a very large distance as compared to its size**.
- Radial fields are considered **non-uniform** fields.
- Hence g is different depending on how far you are from the centre of mass of the sphere



- Newton's Law of Gravitation states that **the gravitational force between two point masses is proportional to the product of the masses and inversely proportional to the square of their separation**.
- This can be written as

$$F_G = \frac{G m_1 m_2}{r^2}$$



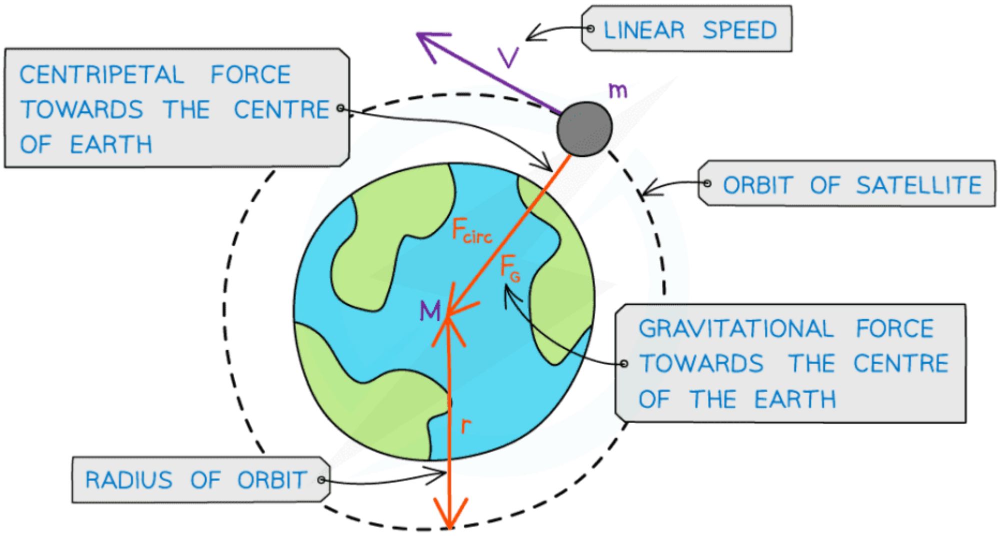
- Here G is Newton's gravitational constant 6.67×10^{-11} Nm^2kg^{-2}
- This value will be given in exam.
- In order for a planet to stay in orbit around the sun (as oppose to falling into the sun!) the planet must travel in a circular orbit around the sun in order for **centripetal force** to balance the **gravitational force**

$$F_C = F_G$$

$$\frac{m_2 v_t^2}{r} = \frac{G m_1 m_2}{r^2}$$

$$v_t^2 = \frac{G m_1}{r}$$

- The equation above proves that all planets travel at same tangential speed (v_t) around the sun since the speed is only dependent on the mass of the sun (m_1)



- Most satellites orbiting the earth follow a **geostationary orbit**.
- The criteria for geostationary orbit are:
  -Remains directly above the equator
  -Moves from west to east (same direction as the Earth spins)
  -Has an orbital time period equal to Earth's rotational period of 24 hours

13.3 Gravitational field of a point mass

Candidates should be able to:

1 derive, from Newton's law of gravitation and the definition of gravitational field, the equation $g = GM/r^2$ for the gravitational field strength due to a point mass

2 recall and use $g = GM/r^2$

3 understand why g is approximately constant for small changes in height near the Earth's surface

- In A levels the candidate must be able to derive the gravitational field equation

$$\boldsymbol{g = \frac{Gm_1}{r^2}}$$

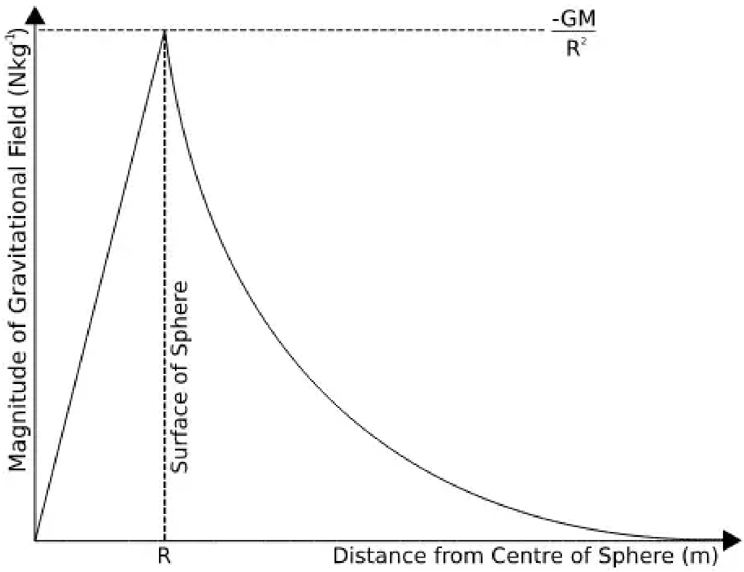
- To derive the equation above first take Newton's law of gravitation force equation

$$F_G = \frac{Gm_1m_2}{r^2}$$

And substitute F_G with

$$F_G = m_2g$$

- **g** here is the same as acceleration due to gravity that you have been using!
- The SI unit is in **N kg^{-1}** or **ms^{-1}**
- Based on the gravitational field equation, g is directly proportional to the **mass of the planet (m_1)** and inversely proportional to the **square of the radius of the planet r^2**



- The value of g changes very little for small changes in height near the surface of the Earth (9.81 ms^{-2}).
- This is because any height change is very small compared to the **radius of the earth (r)**

13.4 Gravitational potential

Candidates should be able to:

1 define gravitational potential at a point as the work done per unit mass in bringing a small test mass from infinity to the point

2 use $\phi = -GM/r$ for the gravitational potential in the field due to a point mass

3 understand how the concept of gravitational potential leads to the gravitational potential energy of two point masses and use $E_p = -GMm/r$

- Recall that gravitational potential energy (GPE) is given by

$$GPE = mgh$$

- It is defined as **the energy an object possess due to its position in a gravitational field**

- We can replace the **height (h)** with the **distance from the center of the earth (r)** and **mass (m)** with the **mass of the object above the earth (m_2)**

$$GPE = m_2 g r$$

- Gravitational potential (ϕ) is defined **work done per unit mass in bringing a test mass from infinity to a defined point**
- So basically, gravitational potential (ϕ) is just GPE per kg^{-1} (GPE/mass)!
- The SI unit is **J kg^{-1}**
- Divide the equation above with m_2

$$\phi = gr$$

Recall that

$$g = \frac{Gm_1}{r^2}$$

Substituting into the above equation we get

$$\phi = \frac{-Gm_1}{r}$$

(Gravitational potential is always negative because it is defined as having a value of zero at infinity and since gravitational force is attractive work must be done on a mass to reach infinity)

The equation for GPE of two-point masses m_1 and m_2 can thus be written as

$$GPE = \frac{Gm_1m_2}{r}$$

If the object was initially at r_1 from the center of the Earth and then moved further to r_2 both GPE and ϕ would be

$$\Delta GPE = Gm_1m_2\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\Delta\phi = Gm_1\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$