15 Ideal gases

15.1 The mole

Candidates should be able to:	
1	understand that amount of substance is an SI base quantity with the base unit mol
2	use molar quantities where one mole of any substance is the amount containing a number of particles of that substance equal to the Avogadro constant N_A

- In thermodynamic, the amount of substance is measured in the SI unit **mole**.
- Mole is defined as **the SI base unit of an 'amount of substance'. It is the amount containing as many particles (e.g., atoms or molecules) as there are atoms in 12 g of carbon-12.**
- The candidate should know from AS that the **atomic mass unit (u)** is equivalent to **1.66×10^{-27} kg**
- A carbon-12 atom has a mass of **12u** (6 protons and 6 neutrons) or **$12 \times 1.66 \times 10^{-27}$ kg**
- Hence

$$1\ mole = \frac{0.012}{1 \cdot 99 \times 10^{-26}} = 6 \cdot 02 \times 10^{23}\ molecules$$

- The Avogardo's constant (N_A) is defined as
  The number of atoms of carbon-12 in 12 g of carbon-12; equal to 6.02×10^{23} mol^{-1}

15.2 Equation of state

Candidates should be able to:	
1	understand that a gas obeying $pV \propto T$, where T is the thermodynamic temperature, is known as an ideal gas
2	recall and use the equation of state for an ideal gas expressed as $pV = nRT$, where n = amount of substance (number of moles) and as $pV = NkT$, where N = number of molecules
3	recall that the Boltzmann constant k is given by $k = R / N_A$

- Any gas that follows the relationship $pV \propto T$ is an **ideal gas**.
- Here p is pressure in Pa, V is the volume of the gas in m^3 and T is temperature in Kelvin.
- Recall that Boyle's Law states that pressure (p) is inversely proportional to volume (V) assuming temperature is constant
- The equation given is

$$P_1V_1 = P_2V_2$$

- Charles' Law states that volume (V) is directly proportional to temperature (T)

- The equation used is

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

- Pressure Law states that pressure (p) is directly proportional to temperature (T).
- The equation used is

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

- Mnemonics time! In order to remember which variable is proportional to which just use
  -Boyle's Law: **B**oy's like to **P**lay **V**ideo games
  -Charles' Law: **C**harlie Brown is a **TV** show
  -Pressure Law: …..
- Remember to use Kelvin and not Celsius in temperature!
- The equation of state for an ideal gas (or the ideal gas equation) can be expressed as:

  pV = nRT

  The equation can also be rewritten as

  pV = NkT

  Here
  -n is the number of moles
  -N is the number of molecules
  -R is the molar gas constant (8.3144598 m^2 kg s^{-2} K^{-1} mol^{-1})
  -k is Boltzmann's constant which is given by k = R/N_A (1.38064852 × 10^{-23} J K^{-1})

- An ideal gas is therefore defined as **a gas which obeys the equation of state pV = nRT at all pressures, volumes and temperatures.**

15.3 Kinetic theory of gases

Candidates should be able to:

1 state the basic assumptions of the kinetic theory of gases

2 explain how molecular movement causes the pressure exerted by a gas and derive and use the relationship $pV = \frac{1}{3}Nm\langle c^2\rangle$, where $\langle c^2\rangle$ is the mean-square speed (a simple model considering one-dimensional collisions and then extending to three dimensions using $\frac{1}{3}\langle c^2\rangle = \langle c_x^2\rangle$ is sufficient)

3 understand that the root-mean-square speed $c_{r.m.s.}$ is given by $\sqrt{\langle c^2\rangle}$

4 compare $pV = \frac{1}{3}Nm\langle c^2\rangle$ with $pV = NkT$ to deduce that the average translational kinetic energy of a molecule is $\frac{3}{2}kT$, and recall and use this expression

- The kinetic theory of gas assumes the following:
  -Molecules of gas behave as identical, hard, perfectly elastic spheres
  -The volume of the molecules is negligible compared to the volume of the container
  -The time of a collision is negligible compared to the time between collisions
  -There are no forces of attraction or repulsion between the molecules
  -The molecules are in continuous random motion
- The pressure of an ideal gas equation includes the **mean square speed** of the particle:

$$\langle c^2\rangle$$

  Here c = average speed of gas particles

- The unit for mean square speed is $\mathbf{m^2s^{-2}}$
- In order to calculate the **average speed** of the particles in a gas, take the square root of the mean square speed:

$$\sqrt{\langle c^2\rangle} = c_{rms}$$

- The unit for c_{rms} is $\mathbf{ms^{-1}}$
- The kinetic Theory of Gases equation is given by

$$pV = \frac{1}{3}Nm\langle c^2\rangle$$

  Where

  -p = pressure (Pa)

  -V = volume (m^3)

-N = number of molecules

-m = mass of one molecule of gas (kg)

-$\langle c^2 \rangle$ = mean square speed of the molecules (ms^{-1})

- On top of being able to apply the equation above, the candidate is expected to know how to derive the kinetic Theory of Gases equation as well:

  -**Step 1:** Find the change in momentum as a single molecule hits a wall perpendicularly

$$\Delta p = -mc - (+mc) = -2mc$$

  -**Step 2:** Calculate the number of collisions per second by the molecule on a wall

  Assume that a gas molecule has to travel from one end of a container to the other end (l). When it bounces after collision back to initial position, the total distance travelled would be 2l. Using

$$speed = \frac{distance}{time}$$

$$Time\ between\ collisions = \frac{distance}{speed} = \frac{2l}{c}$$

  -**Step 3:** Find the change in momentum per second

  Recall

Force = rate of change of momentum

$$\frac{\Delta p}{\Delta t} = \frac{2mc}{\frac{2l}{c}} = \frac{mc^2}{l}$$

  -**Step 4:** Calculate the total pressure from N molecules

  Assume the area of the wall that the molecule collides with is l^2 and using

$$Pressure\ p = \frac{Force}{Area} = \frac{\frac{mc^2}{l}}{l^2} = \frac{mc^2}{l^3}$$

The equation above assumes only one molecule collides with the wall of a container. Hence the equation above is the pressure from one molecule. The total pressure from N molecules can therefore be calculated with

$$Pressure\ p = \frac{Nmc^2}{l^3}$$

Since different molecules have different velocity, we will need to use the mean squared speed $<c^2>$ instead of c^2. The pressure is now

$$Pressure\ p = \frac{Nm < c2 >}{l^3}$$

-**Step 5:** Consider the effect of the molecule moving in 3D space

The previous derivation only took into account the molecules traveling in 1 dimension. Consider the other 2 dimensions, the actual c^2 can be determined using Pythagoras' theorem

$$c^2 = c_x^2 + c_y^2 + c_z^2$$

Assuming that

$$<c_x^2> = <c_y^2> = <c_z^2>$$

Therefore $<c_x^2>$ can be defined as

$$<c_x^2> = 1/3 <c^2>$$

Since l^3 is equal to the volume of the container (V), substituting back into the pressure equation we get

$$pV = 1/3Nm < c2 >$$

- Recall the ideal gas equation

$$pV = NkT$$

Hence

$$NkT = 1/3Nm < \mathrm{c2} >$$

N will cancel out

$$kT = 1/3m < \mathrm{c2} >$$

$$3kT = m < \mathrm{c2} >$$

Multiplying both sides with ½ gets you

$$3/2kT = 1/2m < \mathrm{c2} >$$

Since ½ mc^2 is equal to the kinetic energy of the molecule of an ideal gas we get

$$E_K = 3/2\ kT$$