2 Kinematics

2.1 Equations of motion

Candidates should be able to:

1 define and use distance, displacement, speed, velocity and acceleration

2 use graphical methods to represent distance, displacement, speed, velocity and acceleration

3 determine displacement from the area under a velocity–time graph

4 determine velocity using the gradient of a displacement–time graph

5 determine acceleration using the gradient of a velocity–time graph

6 derive, from the definitions of velocity and acceleration, equations that represent uniformly accelerated motion in a straight line

7 solve problems using equations that represent uniformly accelerated motion in a straight line, including the motion of bodies falling in a uniform gravitational field without air resistance

8 describe an experiment to determine the acceleration of free fall using a falling object

9 describe and explain motion due to a uniform velocity in one direction and a uniform acceleration in a perpendicular direction

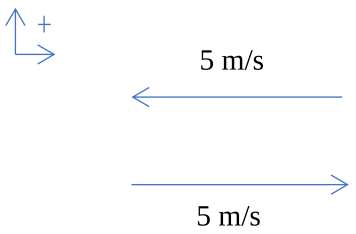
- **Speed** is the total **distance** travelled per unit time (ms^{-1})
- Since **distance** is a **scalar**, **speed** is a scalar
- **Velocity** is the rate of change of **displacement** of an object (ms^{-1})
- Since **displacement** is a **vector**, **velocity** is a vector

$$v = \frac{s}{t}$$

v is the speed, *s* is the displacement and *t* is the time taken

Note: In velocity the positive/ negative sign indicates direction.

- Example



Speed of top arrow: 5 ms^{-1}

Velocity of top arrow: -5 ms^{-1}

Speed of bottom arrow: 5 ms^{-1}

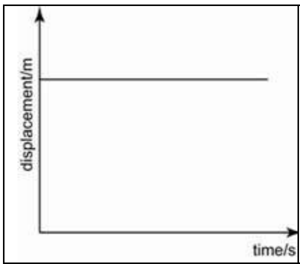
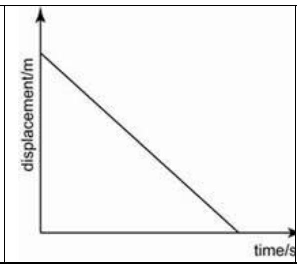
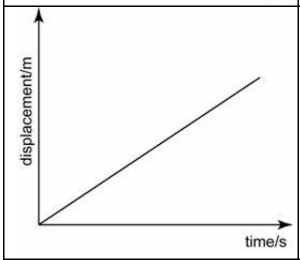
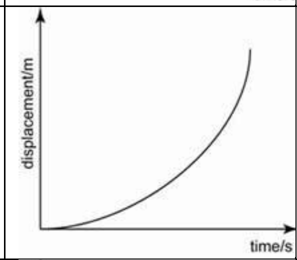
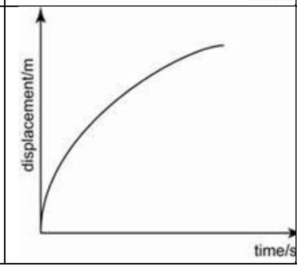
Velocity of bottom arrow: 5 ms^{-1}

- **Acceleration** is the rate of change of **velocity** of an object (ms^{-2})
- **Acceleration** is a **vector**.

$$a = \frac{v - u}{t}$$

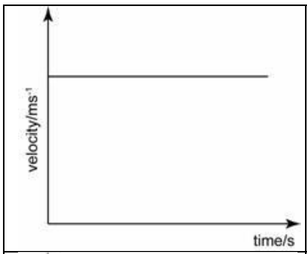
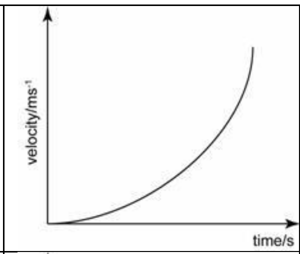
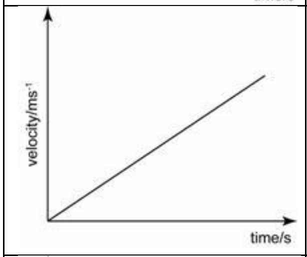
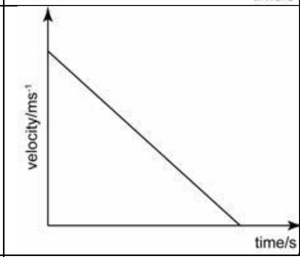
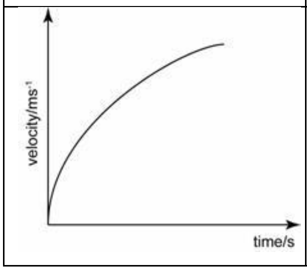
- There are **three** types of motion graphs to represent displacement, velocity and acceleration.
- The three graphs are **displacement-time graphs, velocity-time graphs and acceleration-time graphs**
- You most likely learned the first two graphs in IGCSE or SPM

<u>Displacement-time graphs</u>

- **slope** equals **velocity**
- the **y-intercept** equals the **initial displacement**
- a **straight** line represents a **constant** velocity
- a **curved** line represents an **acceleration**
- a **positive slope** represents motion in the **positive direction**
- a **negative slope** represents motion in the **negative direction**
- a **zero** slope (horizontal line) represents a state of **rest**
- the area under the curve is meaningless

Velocity-time graphs

- **slope** equals **acceleration**
- the **y-intercept** equals the **initial velocity**
- a **straight** line represents **uniform** acceleration
- a **curved** line represents **non-uniform acceleration**
- a **positive** slope represents an **increase** in **velocity** in the **positive direction**
- a **negative** slope represents an **increase** in **velocity** in the **negative direction**
- a **zero** slope (horizontal line) represents motion with **constant velocity**
- the **area** under the curve equals the **change** in **displacement**

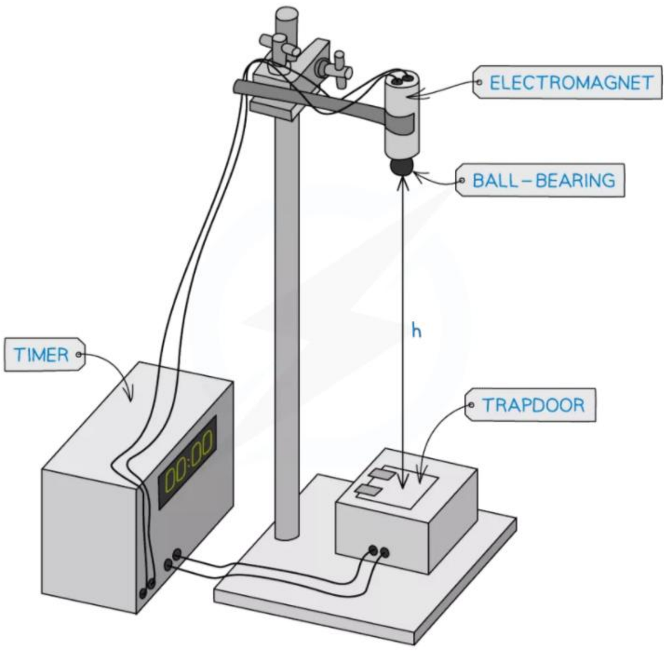
- What about acceleration-time graph?
- On an **acceleration-time graph**…
- slope is meaningless
- the y-intercept equals the initial acceleration
- a zero slope (horizontal line) represents an object undergoing constant acceleration
- the area under the curve equals the change in velocity

- **Kinematic equations** of motion are a set of **four** equations which can describe any objects moving with **constant acceleration**
- The four equations are:

  $v = u + at$
  $d = 0.5(v+u)t$
  $d = ut + 0.5at^2$
  $v^2 = u^2+2ad$

  For A-levels you must know how they are derived

- Below is a description of an experiment to determine acceleration of free fall using a falling object
- **Apparatus**
  Metre rule, ball bearing, electromagnet, electronic timer, trapdoor

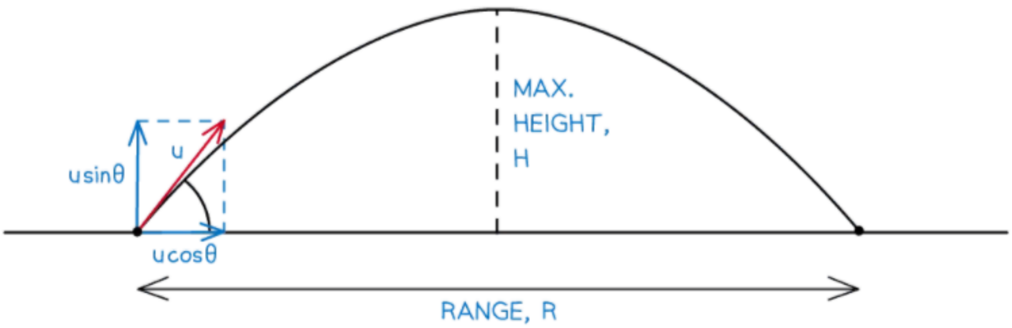


- **Method**
  i) When the current to the magnet switches off, the ball drops and the timer starts.
  ii) When the ball hits the trapdoor, the timer stops.
  iii) The reading on the timer indicates the time it takes for the ball to fall a distance, *h*.
  iv) This procedure is repeated several times for different values of *h*, in order to reduce random error.
  v) The distance, *h*, can be measured using a metre rule as it would be preferable to use for distances between 20 cm – 1 m.

- **Analysing data**
  The known quantities are
  Displacement $s = h$
  Time taken = t
  Initial velocity $u = 0$
  Acceleration $a = g$
  The equation that links these quantities is
  $s = ut + \frac{1}{2}at^2$
  $h = \frac{1}{2}gt^2$
  Using this equation, deduce g from the gradient of the graph of h against t^2

- <u>Sources of error</u>
  Systematic error: residue magnetism after the electromagnet is switched off may cause the time to be recorded as longer than it should be
  Random error: large uncertainty in distance from using a metre rule with a precision of 1mm, or from parallax error

- For A-levels it is not sufficient to know about motion only in 1 dimension alone. The candidate must be able to solve motion in 2 dimensions as well.
- An object undergoing projectile motion consists of 2 components; **vertical** and **horizontal.**
- The key terms in solving projectile motion problems are
  Time of flight: how long the projectile is in the air
  Maximum height attained: The height at which the projectile is momentarily at rest
  Range: The horizontal distance travelled by the projectile
- From the diagram below, it can be inferred that to attain maximum **range**, the optimal angle of flight is **45^0**.



- E.g. of a projectile motion question based on the diagram above:
- A cannon ball is fired at angle of 45°. Assume the initial velocity of a cannonball u = 10 m/s. Find the range R
- **Step 1 Find the time of flight**

  Initial vertical velocity u_V = u sin 45 = 7.07 m/s
  Final vertical velocity v_V = 0 m/s (the cannon ball has to stop in midair before it comes down again!)
  g = -9.81 m/s^2 (negative because acceleration is opposite to the direction of motion!)

  Applying kinematics equation, the time of flight would then be

  $v_V = u_V + gt$
  t = (0-7.07)/-9.81 = 0.72 seconds
  time of flight = 2t = 2x0.72 = 1.44 seconds (time to go up and down)

- **Step 2 Find total range, R**

  Initial horizontal velocity, u_h = u cos 45 = 7.07 m/s
  Final horizontal velocity, v_h = 0 m/s
  Time of flight (from step 1) = 0.72 seconds

  Applying kinematics equation

  $R = 0.5x(v_h+u_h) \times$ time of flight
  $R = 0.5x(7.07+0) \times 1.44$
  R = **5.09 meters**